KEPLER'S LAWS: DETERMINE TRAPPIST-1'S HABITABLE ZONE

TASK 1: REARRANGE KEPLER'S 3RD LAW

You will be using Kepler's 3rd Law to find the orbital distance for each of the exoplanets in the TRAPPIST-1 system.

Rearrange the equation to make a^3 the subject.

$$T^2 = \frac{4\pi^2}{GM} a^3$$

T = orbital period (s)
a = semi-major axis (m)
G = gravitational constant ($G = 6.67 \times 10^{-11}$ m^3 kg^{-1}s^{-2})
M = mass of the star TRAPPIST-1 ($M = 1.72 \times 10^{29}$ kg)

In this case, the semi-major axis (a) is also the orbital distance as we are assuming an eccentricity of 0.

Step 1
Multiply both sides by GM

$$T^2 GM = 4\pi^2 a^3$$

Step 2
Divide both sides by $4\pi^2$

$$\frac{T^2 GM}{4\pi^2} = a^3$$

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TASK 2: CALCULATE THE ORBITAL DISTANCES

Work out the value of a^3 based on the orbital period of each of the exoplanets in the TRAPPIST-1 system (data is given in the data table).

Cube root each answer to find a, the orbital distance, in metres (m).

Lastly, convert each answer into Astronomical Units (AU).

- 1 AU = 1.5×10^{11} m
- Express your value of a to 3 significant figures

Exoplanet	a^3 (m^3)	a (m)	a (AU)
TRAPPIST-1b	4.91×10^{27}	1.70×10^{9}	**0.0113**
TRAPPIST-1c	1.27×10^{28}	2.33×10^{9}	**0.0156**
TRAPPIST-1d	3.56×10^{28}	3.29×10^{9}	**0.0219**
TRAPPIST-1e	8.07×10^{28}	4.32×10^{9}	**0.0288**
TRAPPIST-1f	1.84×10^{29}	5.68×10^{9}	**0.0379**
TRAPPIST-1g	3.31×10^{29}	6.92×10^{9}	**0.0461**
TRAPPIST-1h	7.64×10^{29}	9.14×10^{9}	**0.0610**

Note: the last significant figure in your answers may be different depending on how many significant figures were carried through the equations and how many significant figures of pi (π) were used.

Why might these values differ from the true orbital distance of each exoplanet?

We have assumed an eccentricity of 0. Though each exoplanet has an orbital eccentricity close to 0, it isn't exactly 0.

TASK 3: FIND THE HABITABLE ZONE OF TRAPPIST-1

The equations below can be used to calculate the minimum and maximum distance (d_{min} and d_{max}) from the TRAPPIST-1 star where liquid water could potentially exist.

They give d in units of AU.

Use the value of L_{star} given below the equations to work out these distances.

- Express your answers to 3 significant figures

$$d_{min} = \sqrt{\frac{L_{Star}}{0.9}} \qquad d_{max} = \sqrt{\frac{L_{Star}}{0.23}}$$

L_{Star} = Luminosity of the star TRAPPIST-1 in solar luminosities

(L_{Star} = 0.000522 $L_{\odot}$)

d_{min} = **0.0241 AU**	d_{max} = **0.0476 AU**

Note: the constants (0.9 and 0.23) in each equation assumes the exoplanets have an atmosphere like Earth's and that their surfaces reflect the same amount of light.

There is also the assumption that the star TRAPPIST-1 is similar to the Sun.

The difference between the constants comes from d_{min} using a temperature of the boiling point of water and d_{max} using a temperature of the freezing point of water.

TASK 4: DETERMINE WHICH EXOPLANETS ARE HABITABLE

The minimum and maximum distances (d_{min} and d_{max}) give the inner and outer edges of TRAPPIST-1's habitable zone.

Look at the orbital distance of each exoplanet.

Which exoplanets in the TRAPPIST-1 system fall within the habitable zone?

TRAPPIST-1e, TRAPPIST-1f, TRAPPIST-1g

Why might these exoplanets **not** have liquid water on them despite being in the habitable zone?

- **We are assuming they have the same atmosphere as Earth. They could actually experience a greater or lesser greenhouse effect depending on the gases in their atmosphere.**
- **We are assuming they reflect the same light as the Earth. Surface structure and texture, atmospheric composition, the presence of clouds, etc. can all effect the amount of light reflected by a planet.**
- **We are assuming that the star TRAPPIST-1 is similar to the Sun. In reality, TRAPPIST-1 is a cool red dwarf much smaller and less luminous than the Sun.**